$$\therefore\ y^2-9=y\frac{B}{A}-3\frac{B}{A}\ ,\ \therefore\ y^2-\frac{B}{A}y=9-3\frac{B}{A}.$$

Completing the square root, $y^2-\frac{B}{A}y+\frac{B^2}{4A^2}=9-3\ \frac{B}{A}+\frac{B^2}{4A^2},\ \therefore\ y=3.$

Substituting value of y in (4) and (5),

$$x-4=c(2-z)\ldots.(9),\ d(x-4)=2-z\ldots.(10).$$

Eliminating $(2-z)$ between (9) and (10), $\frac{x-4}{c}=d(x-4)=x^2-16,$

$$\therefore\ x^2-\frac{x}{c}=16-\frac{4}{c}.\quad \therefore\ x^2-\frac{x}{c}+\frac{1}{4c^2}=16-\frac{4}{c}+\frac{1}{4c^2},\ \therefore\ x=4.$$

Values of x and y in $x^2+y^3+z=45$, gives $z=2$.

$\therefore\ x=4,\ y=3,\ z=2.$

Also solved by *Professor W. F. Bradbury.*

10. Proposed by J. K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pennsylvania.

$$x^2+y^2+w^2+z^2=65\ldots.(1),$$
$$(x+z)^2+(y+w)^2=113\ldots.(2),$$
$$(y+z)^2+(x+w)^2=117\ldots.(3),$$
$$(x+y)^2+(z+w)^2=125\ldots.(4).$$

How many values has each of the four unknown quantities?

Solution by W. F. BRADBURY, A. M., Head-Master Cambridge Latin School, Cambridge, Massachusetts.

From (1) subtract (2), (3), and (4), successively,

$$2xz+2yw=48\ldots.(5),$$
$$2yz+2xw=52\ldots.(6),$$
$$2xy+2zw=60\ldots.(7).$$

Adding (5), (6), (7), and (1), we get,

$$(x+y+z+w)^2=225\ldots.(8),\quad x+y+z+w=\pm15\ldots.(9),$$

Using only + values, $x+z=15-(y+w)\ldots.(10)$.

Substituting in (2), $225-30(y+w)+(y+w)^2+(y+w)^2=113\ldots.(11)$, $(y+w)^2-15(y+w)=-56\ldots.(12)$,

$$y+w=\frac{15}{2}\pm\sqrt{\frac{225}{4}-\frac{224}{4}}=\frac{15}{2}\pm\frac{1}{2}=8,\text{ or }7\ldots.(13).$$

Hence from (10), $x+z=7$, or 8. In like manner, substituting from (9) in (3) and (4), we find $y+z=6$, or 9; $x+w=9$, or 6; $x+y=5$; $z+w=10$.

From these we find, $x=3$, or 2, $y=2$, or 3, $w=6$, or 4, $z=4$, or 6. Using the negative values other answers can be found.

[There are in all 16 values for each of the unknown quantities, arising from the reduced equations $x+y+z+w=\pm15$, $x+y+z+w=\pm5$, $x+y+z-w=\pm1$, $x-y-z+w=\pm3$, as follows:

$$x=\pm6,\ \pm4,\ \pm2,\ \pm3,\ \pm4\tfrac{1}{2},\ \pm5\tfrac{1}{2},\ \pm3\tfrac{1}{2},\ \pm1\tfrac{1}{2}.$$
$$y=\pm4;\ \pm6,\ \pm3,\ \pm2,\ \pm5\tfrac{1}{2},\ \pm4\tfrac{1}{2},\ \pm1\tfrac{1}{2},\ \pm3\tfrac{1}{2}.$$
$$z=\pm2,\ \pm3,\ \pm6,\ \pm4,\ \pm3\tfrac{1}{2},\ \pm1\tfrac{1}{2},\ \pm4\tfrac{1}{2},\ \pm5\tfrac{1}{2}.$$
$$w=\pm3,\ \pm2,\ \pm4,\ \pm6,\ \pm1\tfrac{1}{2},\ \pm3\tfrac{1}{2},\ \pm5\tfrac{1}{2},\ \pm4\tfrac{1}{2}.$$—EDITOR.]

Also solved by Professor *G. B. M. ZERR.*